

ICON-SMART 2023  
Univ. of Sam Ratulangi  
Manado, Indonesia  
2023/09/21, 10:00-10:20



## Graph coverings and their applications

Under the theme: Data Optimization for Instrumentation Physics,  
Food Security, and Energy Sustainability.  
How About AI?

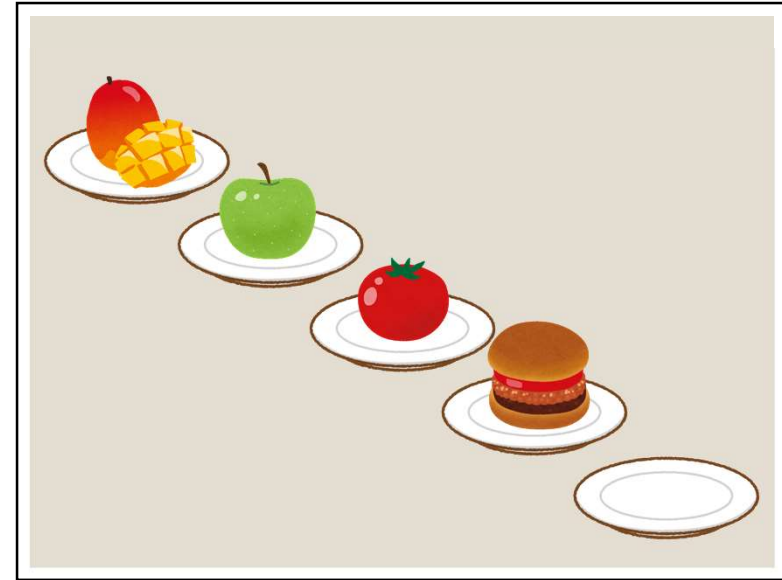
### Seiya NEGAMI

---

Dr. and Professor in honor,  
Yokohama National University, Japan

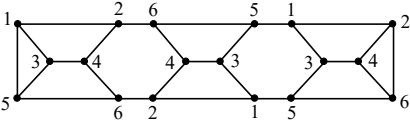
1



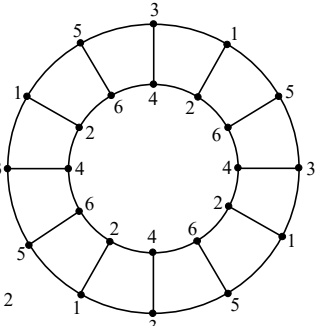
2

## Graph covering...

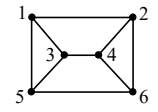
There is a bijection between the neighbors of corresponding vertices.



Covering graph



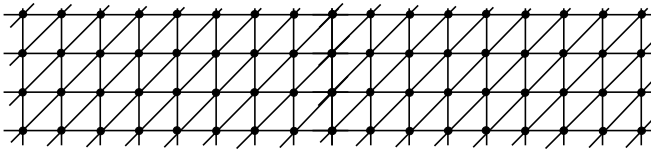
projection

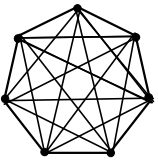


Base graph

3

## A huge graph covers a small graph...





What can we say about a huge graph, looking only a small graph?

+

 $\pi_1(G)$ 

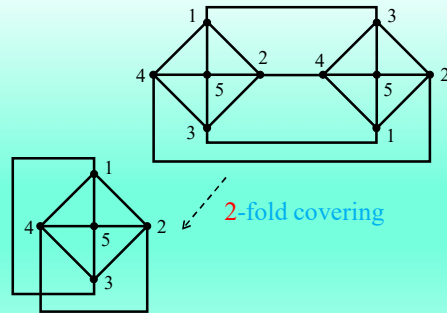
by methods in algebraic topology

Approach to big data analysis using topological graph theory / iCOR4.0, 2019

4

## Planar Cover Conjecture 1986

If a connected graph  $G$  has a finite planar covering, then  $G$  has a 2-fold planar covering.



5

Default (GPT-3.5)

NE Please teach me how to solve Planar Cover Conjecture.



6

Default (GPT-3.5)

NE Please teach me how to solve Planar Cover Conjecture.



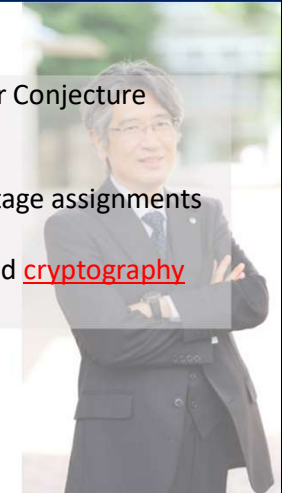
I'm sorry, but as of my last knowledge update in September 2021, there is no widely known mathematical conjecture or problem called the "Planar Cover Conjecture." It's possible that this conjecture or problem has emerged after my last update, or it may not be a well-known or established mathematical problem.

If you could provide more information or context about the Planar Cover Conjecture, I would be happy to help you understand it or explore any related mathematical concepts. Alternatively, if it is a recent development in the field of mathematics, I recommend consulting current mathematical literature or reaching out to experts in the field for the most up-to-date information and solutions.

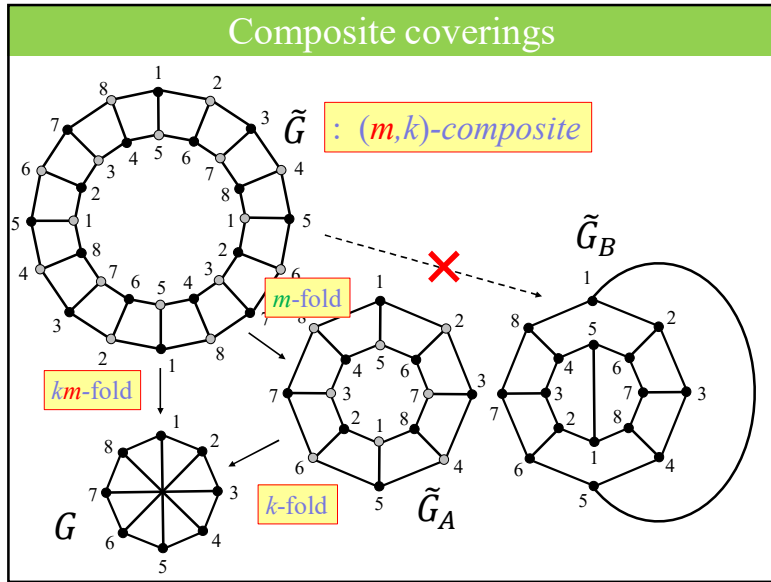
7

## Graph Covering Trilogy

- Another approach to Planar Cover Conjecture focusing on rotation systems to appear in *J. Math. Soc. Japan* (2023)
- Note on graph coverings with voltage assignments *Yokohama Math. J.* 68 (2022), 109-126.
- Composite coverings of graphs and **cryptology** preprint 2023.



8



9

### Intermediate double coverings

**Theorem 1** If  $m$  is an odd number, then  $(m, 2)$ -composite covering has a **unique** double coverings, up to equivalence.  
(Negami 2023)

“Does  $G$  cover  $H$ ?” is NP-complete.

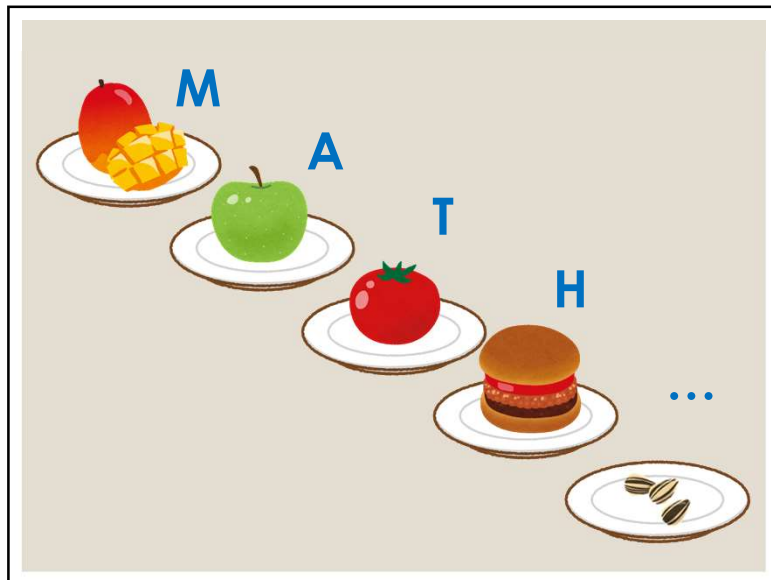
Bodlaender 1989

$\tilde{G}$   
 $\downarrow$   
 $G$

$\swarrow$   
 $m\text{-fold}$   
 $\tilde{G}_A \tilde{G}_B \tilde{G}_C \dots$   
 $\searrow$   
 $2\text{-fold}$

$G \supset T : \text{spanning tree}$

10



11



12